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LETTER TO THE EDITORS

ON THE FLAT PLATE APPROXIMATION TO LAMINAR FREE CONVECTION FROM THE OUTER FACE OF A VERTICAL CYLINDER

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SINCE the appearance of the classical study by Sparrow and Gregg [1], laminar free convection from the outer surface of a vertical cylinder has been treated by a number of investigators. One cornerstone of pertinent analysis is the determination of the suitability of flat plate theory in a given configuration to estimate heat (or mass) transfer rates at the cylindrical surface. Sparrow and Gregg confined their approach to $Pr = 0{\cdot}72$ and $1{\cdot}0$; the 5 per cent criterion (i.e. the criterion which ensures that flat plate analysis is not less accurate than 95 per cent) was given as [2]

$$\frac{\text{Diameter of cylinder, } D}{\text{Length of cylinder, } L} \geqslant \frac{35}{Gr^{\frac{1}{4}}}; Pr = 0{\cdot}72, 1{\cdot}0. \quad (1)$$

Elenbaas [3, 4] and LeFevre and Ede [5] derived more complicated relationships; the latter agree quite closely with the Sparrow–Gregg model for low Prandtl numbers, provided that $(D/L)\,Ra_L^{\frac{1}{4}}$ is larger than 30.

The instance where Pr is rather large is especially important in estimating heat and mass transfer rates in electrolysis of aqueous electrolytes at vertical cylinders. Eigenson's pioneering work [6] suggests that if $Gr_D > 10^6$, flat plate theory can be applied with great precision, regardless of the value of Pr. More recent work by Fujii *et al.* [7, 8] indicates that the larger the Prandtl number, the better the flat plate approximation; however, their analysis does not exceed $Pr = 100$. On the other hand, the theory of free convection from axisymmetric surfaces by Acrivos [9] states that at infinitely large Prandtl numbers the flat plate analysis is completely accurate for a vertical cylinder of any geometry (i.e. D/L ratio).

The purpose of this letter is to present a generalized form of the criterion given by equation (1), which can be applied to any Prandtl number between unity and infinity. The generalized relationship was developed by following the approach of Sparrow and Gregg [1] and Fujii [8]; in so doing, the third set of ordinary differential equations, whose contribution has been found negligible, was removed. The remaining two sets, which constitute a four-point boundary value problem, where solved numerically by applying a simple optimum-search technique to arrive at the initial conditions. Table 1 shows the dependence of the approximation-to-flat-plate criterion

$$\frac{D}{L} \geqslant \frac{3(2)^{\frac{1}{2}}}{R-1}\cdot\frac{\theta_l'(0)}{\theta_l'(0)}\,Gr^{-\frac{1}{4}}; R \triangleq \frac{\overline{Nu}_{\text{cyl}}}{\overline{Nu}_{\text{fp}}} \quad (2)$$

on the magnitude of the Prandtl number. The Table also shows the previous results by Fujii and Sparrow and Gregg; notice the slight disagreement at low Pr. The dependence of D/L on Pr can be expressed by the least-squares relationship

$$\frac{D}{L} \geqslant \frac{1{\cdot}66115}{R-1}\,Pr^{-0{\cdot}2864}\,Gr^{-\frac{1}{4}} \quad (3)$$

with a residual sum of squares of 0·00106 (for $Pr = 1$, the second entry in Table 1 was used). Equation (3) permits the estimation of the accuracy of flat plate theory at any $Pr \geqq 1$ and Gr in the case of a vertical cylinder.

Table 1. The dependence of the approximation criterion on Pr

Pr	$\theta_l'(0)$	$\theta_l'(0)$	$3(2)^{\frac{1}{4}}\dfrac{\theta_l'(0)}{\theta_l'(0)}$	Remarks
1	—	—	1·7500	From [1]
1	−0·5671	−0·2236	1·6730	From [8]
100	−2·1910	−0·2254	0·4364	From [8]
1000	−4·2740	−0·2280	0·2263	
1500	−4·7370	−0·2290	0·2051	
2000	−5·0710	−0·2300	0·1925	

The exponent of the Prandtl number in equation (3) is very close to the intuitively expected value of 0·25. The numerical accuracy of the solution of the problem depends on how closely the boundary conditions are satisfied. This information is not available in [1] and [8]; in the current work, the accepted solutions satisfy the "near-zero" conditions: $f_0' \leqslant 0{\cdot}001$; $f_1' \leqslant 0{\cdot}001$; $\theta_0 \leqslant 0{\cdot}0001$ at the final value of η, estimated from the Levich equation [10] $\eta = 1{\cdot}438/Pr^{\frac{1}{4}}$. This relationship is correct only for a flat plate, but if R is slightly larger than unity, the hereby computed value of η_F is acceptable instead of the numerically indeterminate condition of $\eta \to \infty$. Furthermore, the value of the exponent is 0·233, if computed using the last three entries in Table 1. Thus, the exact form of equation (8) is most likely

$$\frac{D}{L} \geqslant \text{const.}\, Ra^{-\frac{1}{4}}; R \text{ fixed} \quad (4)$$

but from a practical point of view, pursuance of an exact numerical solution is not particularly important.

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